Emission of a Photon in a Multimode Cavity

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Outline

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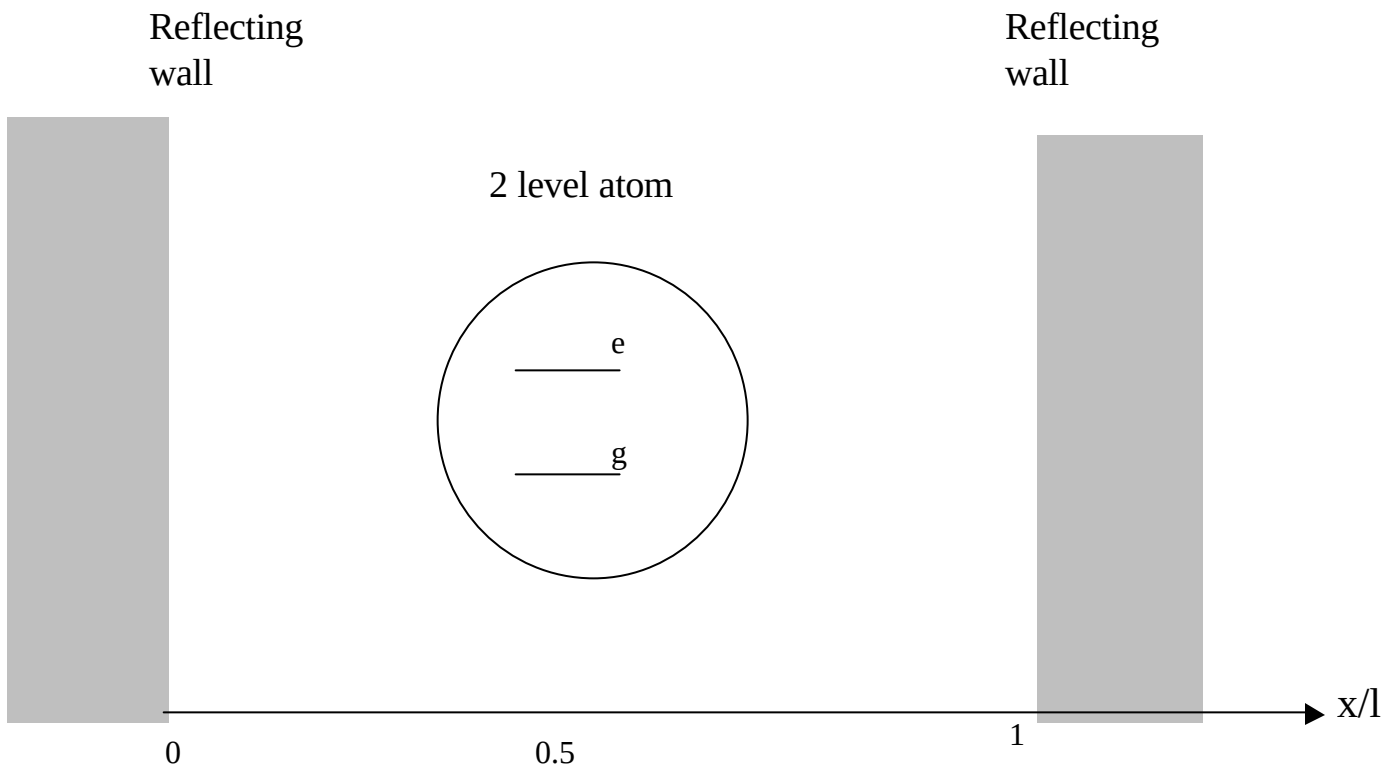
Introduction

-two level atom in a multimode cavity with perfectly reflecting walls

-atom initially in excited state, cavity modes in ground or vacuum state

-atom is coupled to all modes of the cavity and can thus emit a photon into any cavity mode

-spatial localization of electric field squared and number density will be examined


Reflecting
wall
Reflecting
wall
2 level atom
e
g
x/l
0
0.5
1

Theory

- Based on study of $\langle E^2 \rangle$ in the cavity (Ligare/Oliveri, Am. J. Phys. 70, 58 (2002))
- Electromagnetic(EM) modes

  $E(x) \sim \sin(j\pi x/l)$, $w_j = jpc/l, \Delta_c = pc/l$
- Denote mode that matches atomic resonance by j_0
- Zero photons $\rightarrow |e,0,0....\rangle \rightarrow |e,0\rangle$
- One photon $\rightarrow |g,0,0,n_j = 1,0,0\rangle \rightarrow |g, j\rangle$

The Hamiltonian

$$H_f = \sum_{j=1}^{\infty} \hbar w_j \left(a_j^{\dagger} a_j + 1/2 \right)$$

$$H_a = \hbar w_{eg} s_3$$

$$H_i = \mathbf{E} \cdot \mathbf{d}$$

$$\approx \sum_j h(\Omega_j s_+ a_j + \Omega_j^* s_- a_j^t) \sin[jp x_a / l]$$

$$\Omega_j \equiv d_{eg} \left(w_j / 2he_0 V \right)^{1/2}$$

$$H = H_a + H_i + H_f$$

Eigenvalue Problem

$$H\left|E^{(q)}\right\rangle = E^{(q)}\left|E^{(q)}\right\rangle$$

Energy eigenstates: $\left|E^{(q)}\right\rangle = d_e^{(q)}\left|e,0\right\rangle + \sum_j d_{g,j}^{(q)}\left|g,\, j\right\rangle$

Solve for $d_e^{(q)}$ and $d_{g,j}^{(q)}$.

Wavefunction

$$\left|y\ (t)\right\rangle = \sum_{q} e^{-iE_q t/h} (d_e^{(q)} \left|e,0\right\rangle + \sum_{j} d_{g,j}^{(q)} \left|g,\ j\right\rangle) d_e^{(q)*}$$

$$\left\langle e,0 \middle| E_q \right\rangle = d_e^{(q)}$$

$$\left\langle g,\ j \middle| E_q \right\rangle = d_{g,j}^{(q)}$$

Electric field squared

$$\left\langle y\left(t\right)\left|E^{2}\right|y\left(t\right)\right\rangle = 2\left|\sum_{j,q} C_{j} d_{e}^{(q)*} d_{g,j}^{(q)} e^{-iE_{q}t/h} \sin\left[\, jp\, x/l\,\right]\right|^{2}$$

$$C_{j} = \sqrt{hw_{j}/e_{0}V}$$

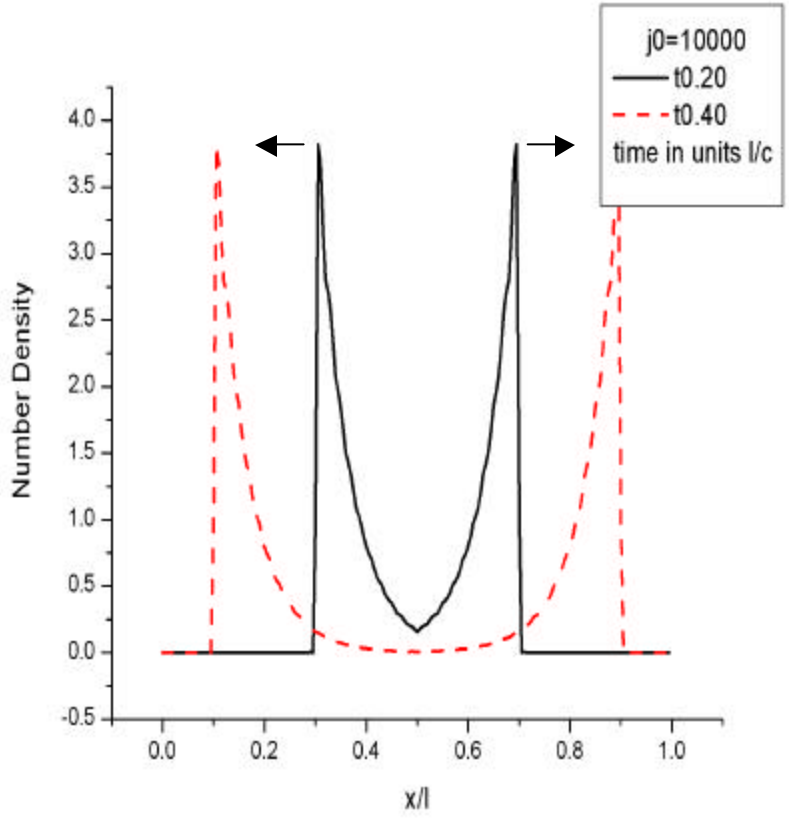
Number Density

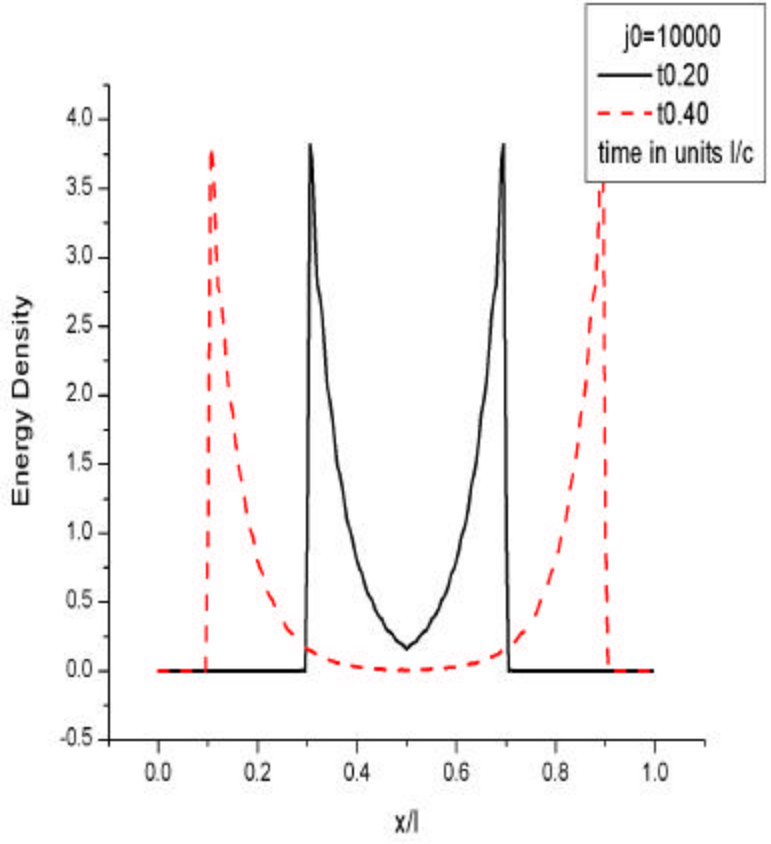
$$2A^2 \left| \sum_{j,q} d_e^{(q)*} d_{g,j}^{(q)} e^{-iE_q t/h} \sin[jp\,x/l] \right|^2$$

Results

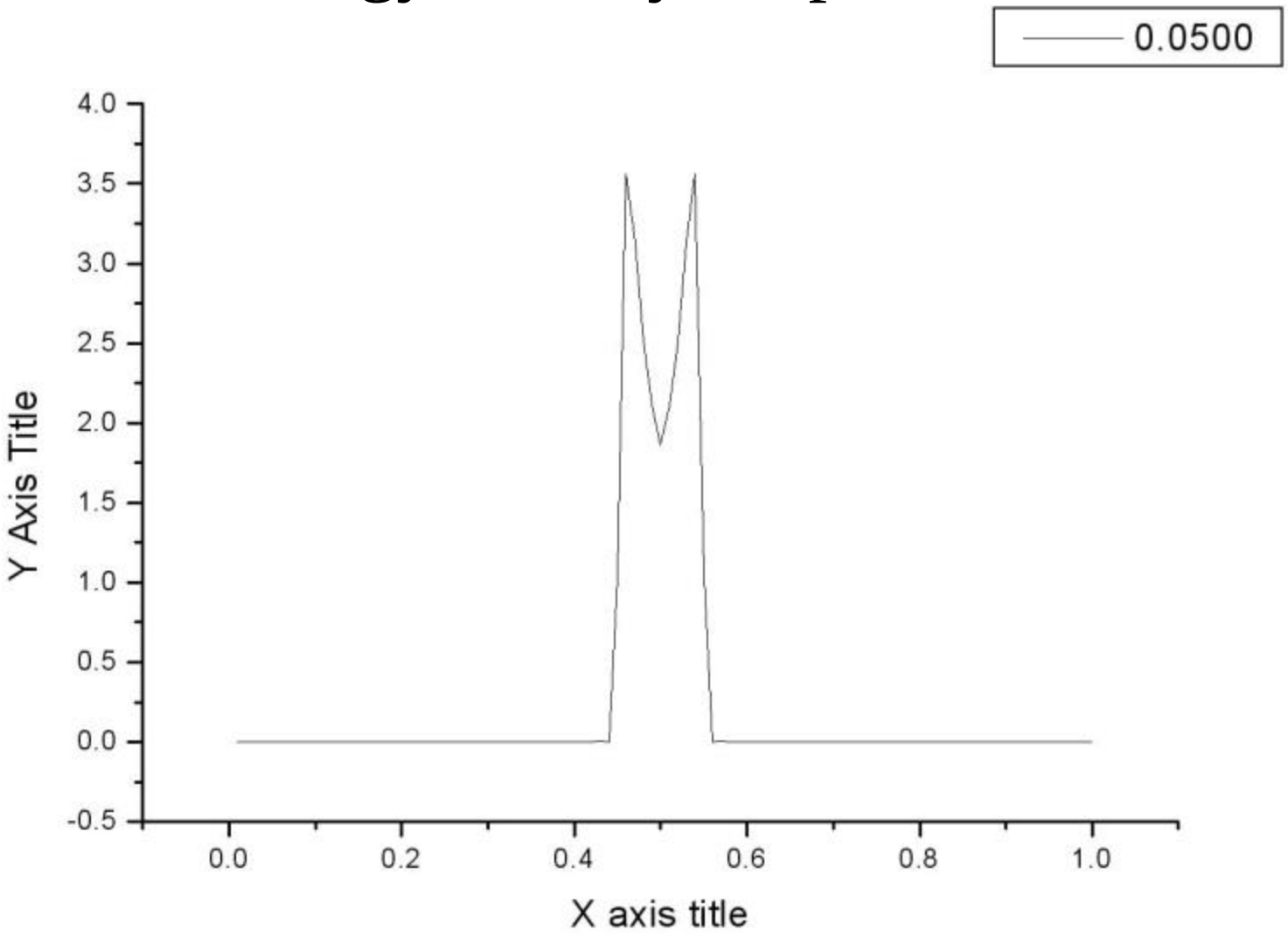
- We examine results for energy and number density for low resonant mode number and high resonant mode number

High Resonant Mode Number (j0=10000)

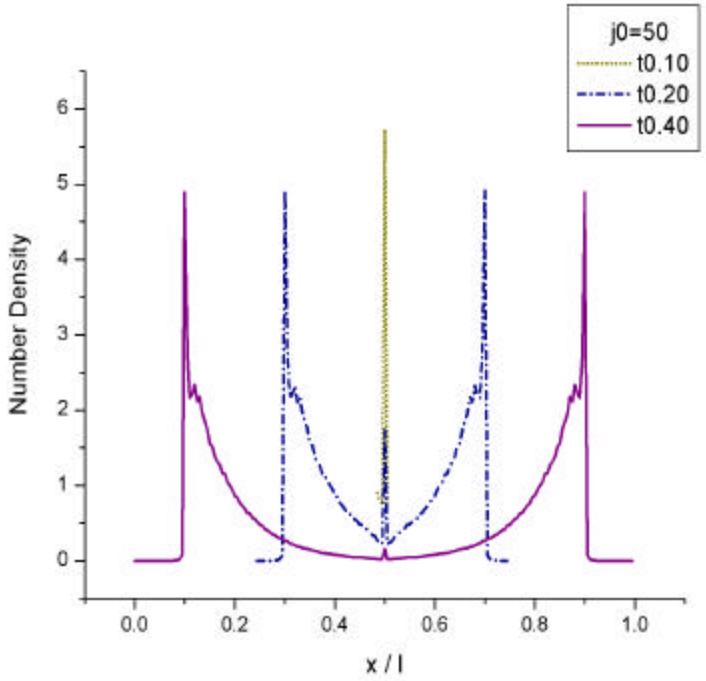


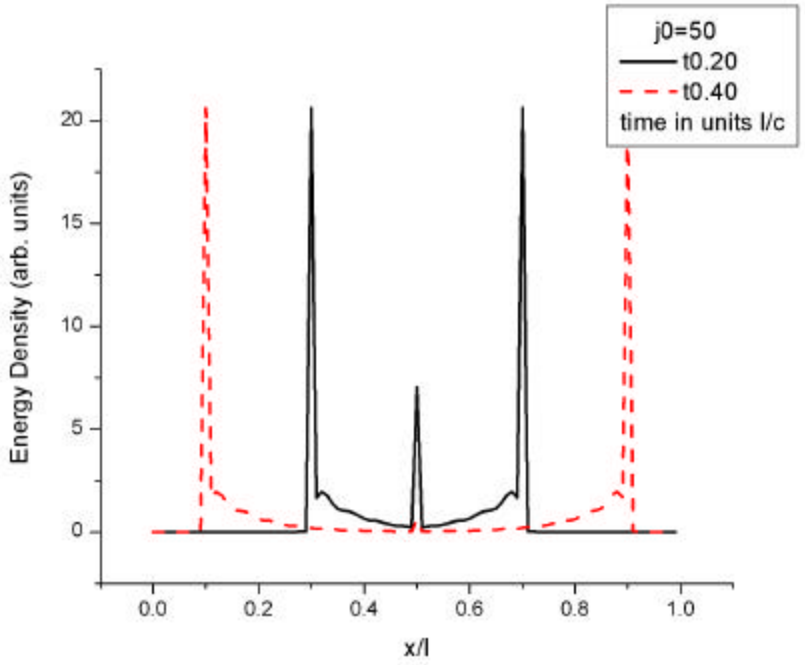


Energy Density vs. position

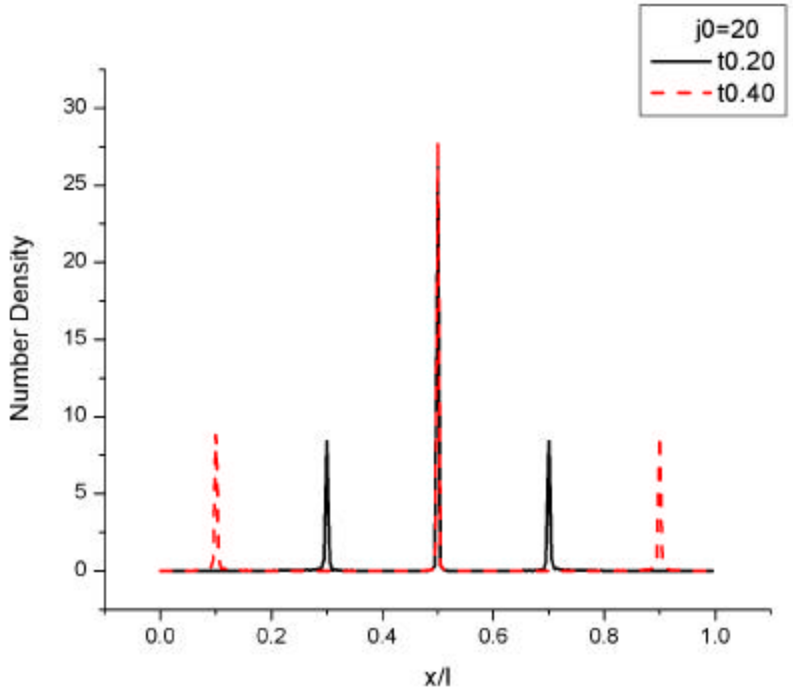


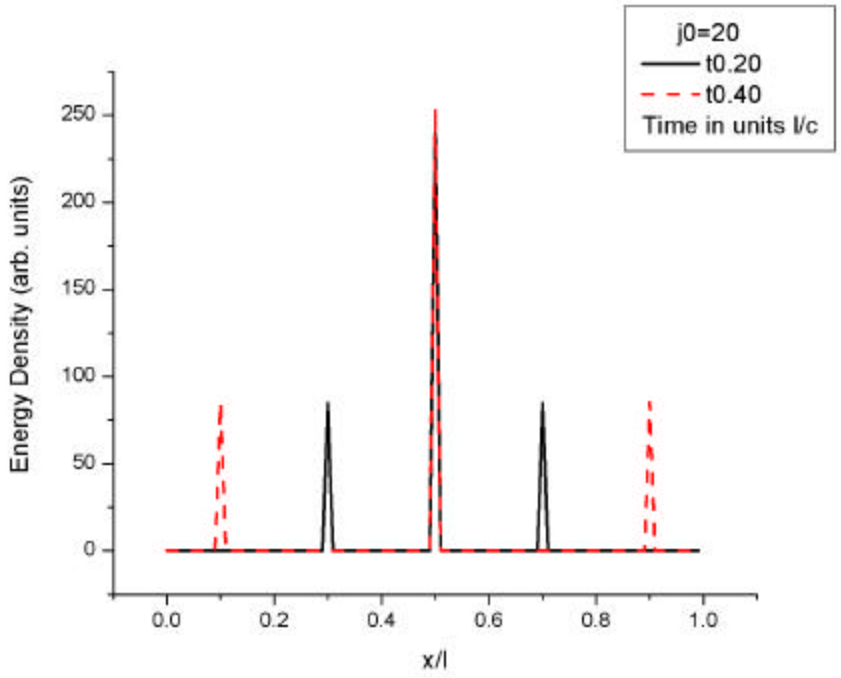
Low Resonant Mode, $j_0 = 50$



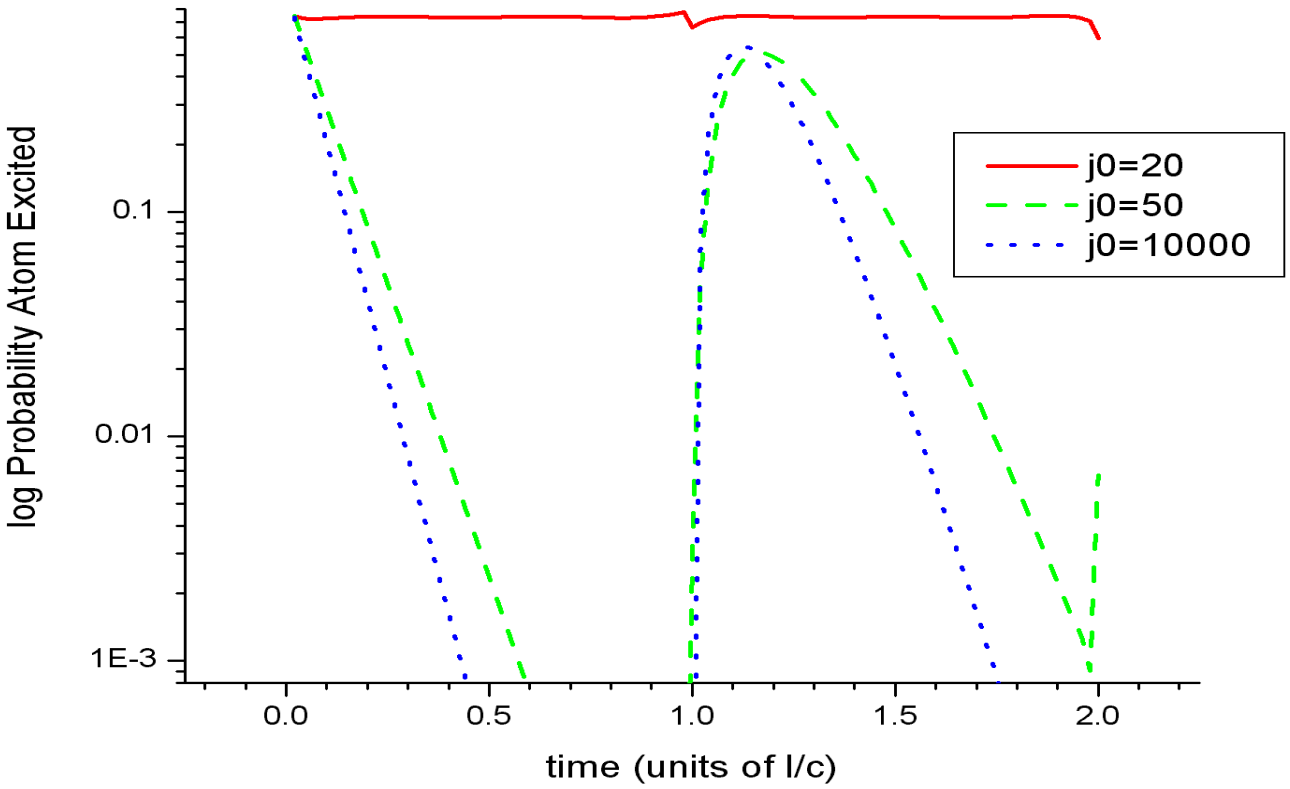


Low Resonant Mode, j_0=20





Atomic Excitation



Conclusion

- Energy density and number density due to spontaneous and stimulated emission for a photon in a cavity have been calculated
- For low j0, number and energy density is not the same shape
- Reduced decay rate is found to be due to photon field at the atom