Photon Matter Interactions in a Cavity: Where is the Photon?

Zachari Medendorp

Lakehead University

(Dated: April 2008)

Abstract

A one dimensional closed form analysis of polariton production in a Fabry-Perot cavity is developed. A simulated comparison is made between the absorption efficiency of a reflected and time reversed emitted dipole field, where is it was found that less then 80% of the reflected field was absorbed in contrast with 100% efficiency of the time reversed field, in ideal situations. The localization of a photon in one dimension is achieved. For a two-level semicondutor in its ground state, the formation of a photon cloud was observed.

Contents

I. INTRODUCTION

I will try to answer the question "where is the photon?" for a 1-photon state. Is the photon's position determined by the energy density or the number density? Since the energy of a photon is $\hbar\omega$ (ω is angular frequency), the electric field goes as $\sqrt{\hbar\omega}$, while the number density omits this factor. We need to consider the photon energy and number density as measured in photon counting experiments. It is often claimed that photon position can't be measured with accuracy better than the order of a wave length. An atom is a small photodetector, much smaller than a wave length, so is this true?

For the acoustic analog, the experiment has been done, so the answer is known. The basic principles behind the acoustic experiment demonstrated by de Rosny and Fink [1] are that there exist two wave equations, one with a source term and one without, both of which give rise to outgoing and incoming spherical waves where the latter is the time reversed

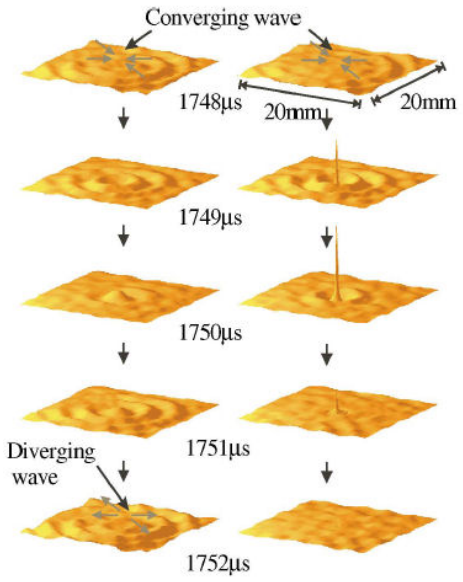

FIG. 1: Experimental Results by de Rosny & Fink: Pictures on the left show acoustic TR without a source;Pictures on the right show acoustic TR with a source.

(TR) version of the former. Initial experiments were done by removing the source when the TR wave was produced. But this is not exactly time reversal of the emission process, since the source, which is a sink when time reversed, is now missing. When the sink is included, sub-wavelength focusing of an acoustic wave is possible.

Experimental results by de Rosny and Fink[1] on acoustic TR with and without a source are shown in Figure 1. Results on the left of the figure show that as the incoming wave approaches the focal point the amplitude of the wave increase until the spot size is of the order of a wavelength. This is consistent with Abbe and Rayleigh's resolution limit which states that the spacial resolution D must satisfy $D \geq 0.4\lambda$. However, after the field focuses it begins to diverge away from the origin, thus breaking the TR process. Comparing this with right side of the figure where the source is still present, it can be noted that the amplitude of the wave continues to rise until the wave front is nearly the size of the source point. If the dimensions of the source are considerably smaller than the wavelength then it would appear that the resolution limit has been beaten, as was the case in this experiment where they achieved a focal spot size of $1/14^{th}$ of the wavelength. So it was found that sub-wavelength detection is possible in this case.

The goal of this thesis work is to see what can be learned about subwavelength photo-detection using computer simulation. For practical reasons and because of the time constraint of an undergraduate thesis, I will stick to a two-level system in one dimension(1D). There is a physical realization of this, a semiconductor in the center of a Fabry Perot cavity with perfectly reflecting walls.

Emission is closely related to absorption and hence detection, in fact it is just emission viewed backwards in time. The TR pulse shape shows how to achieve better photo-detection. This will be investigated further in the theory section on light matter interactions, which contains both qualitative and quantitative descriptions of the TR system and the photon-exciton coupling that exists to produce an exciton-polariton, or just polariton.

A polariton is a quasi-particle formed by the strong coupling of electromagnetic waves with some electric or magnetic dipole that can be excited. A related probelm is the ground state system interacting with vacuum fluctuations. The zero point fluctuations of the field induce absorption and re-emission of virtual photons producing a cloud of virtual photons. The combination of the bare two-level system and photon cloud is what can be referred to as a dressed system [6].

Polariton experiments designed with similar aspects to the system analyzed in this thesis have been performed [9]. The basic setup of the experiment consists of a planar Fabry-Perot cavity whose mirrors are multilayer structures built with alternating layers of two different refractive indexes and the same thickness of $\frac{\lambda}{4}$, where λ is the resonant wavelength of the cavity. This presents a high reflectivity of the resonant mode. The semiconductor, doped quantum wells, are constructed in between the two mirrors, where multiple wells are required to increase the coupling strength of the exciton-photon interaction. Rabi oscillations, fluctuations between two states (ie. photon and exciton states), are observed and can be interpreted as a polariton as they represent resonantly coupled semicondutor and field oscillators periodically exchanging energy. This produces states which exhibit a splitting in their energy levels. A theoretical analysis of polariton splitting in quantum wells has been done [10] and similar results to mine have been shown [8].

II. THEORY

A. Qualitative Description of Light Matter Interactions

1. Time Reversal of an Electromagnetic Field

The acoustic result [1] of sub-wavelength focusing achieved by time reversing both the wave and the source is intriguing, and reveals that an interesting way of thinking about absorption as the TR emission process. Electromagnetic waves are transverse vectors, so the situation is more complicated. But by taking a look at the way an electric dipole radiates we can get some insight as to what is required to time reverse light back to absorption.

A doughnut-shaped intensity pattern emitted perpendicular to an oscillating dipole is a suitable model and was used by Leuchs [2] to develop a method to focus light to a tighter spot. So in order to increase the level of absorption, one would need to successfully generate the TR version of the electric field such that in the near field it resembles that of this doughnut pattern. Thinking again about the emission by the dipole, it is known that the amplitude of the diverging doughnut shaped wave emitted from the dipole falls off as the inverse of the radial distance in the far field, but at the origin the amplitude is large. Considering this process in reverse, the doughnut shaped field is propagating inwards towards the origin. If the dipole is still at the origin then as the wave approaches the dipole it will produce a non-propagating field in the near field zone by its back action on the field. These evanescent fields are necessary for the wave's amplitude to continue increasing. The amplitude will increase even beyond the point where the wave front is of the order of its wavelength and will only cease when the wave front is of dipole dimensions, if the longitudinal field is included. I say "if the longitudinal field is included" because currently when analysis is done the transverse and longitudinal components are treated separately and it is concluded that the photon cannot be localized to such a small size. But if we were to consider both fields simultaneously then we would get different results. Without the presence of the dipole the increase in amplitude of the incoming wave comes to a stop when the diameter of the wave front is nearly the size of its wavelength. In the former it appears that the resolution limit has been violated, but do longitudinal fields describe photons?

Physically time reversing an EM field cannot be easily done in a similar manner to that of acoustic time reversal as previously stated. One method of TR is based on four-wave mixing

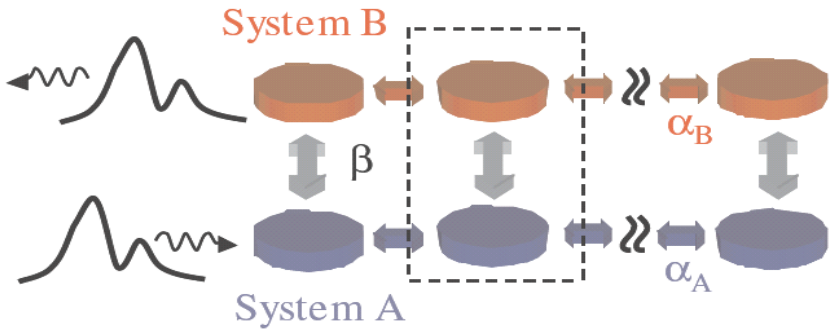

FIG. 2: From Yanik, the incoming wave enters system A where it propagates through the photonic crystal. At the same time the crystal's index of refraction in system A and B are being modulated in such a way that the field couples to system B but with the wave form of the field reflected about the central wave vector. The wave will propagate out from system B as the TR version of the incoming wave.

which involves the interaction of three wave vectors within a nonlinear medium giving rise to a fourth wave vector produced by the scattering of the incident photons. This method of mixing waves is used to produce a phase conjugate mirror which acts like a TR mirror. The 3 mixing waves consists of 2 pump beams and a probe; their interaction produces the back propagating signal wave thus time reversing the probe. This works fine for monochromatic light but for a pulse, phase matching needs to be satisfied over the whole bandwidth which requires the development of more complicated nonlinear media.

A recent method to simulate TR in 1D [3] has been developed with the use of photonic crystals. It has been shown that the spectrum of a photon can be modified when the index of a photonic crystal is modulated. TR is achieved by modulating the index in the crystal and by evanescent coupling between two systems A & B where the wave shape is reproduced but with a center wave vector in the opposite direction, as sketched in Fig. 2.

The important thing here is that one can simulate time reversal by reversing the amplitude of the Fourier components relative to the center frequency. Then the pulse envelope moves backwards. Note that time reversal is not equivalent to reflection at a boundary.

2. *Polaritons in a One-Dimensional Cavity*

The process involved in creating a polariton will be discussed based on the system described above, a semiconductor in a one dimensional cavity. When the system is in its ground state the semiconductor valence band is completely filled and the conduction band, which acts as the excited state, is unoccupied. A single photon with the appropriate energy incident on the semiconductor will excite an electron to occupy the excited state of the system. This creates an exciton in the semiconductor which can now do a number of things. Depending on the initial conditions of the system the electron-hole can recombine, emitting a photon back into the cavity which will propagate away from the semiconductor towards the cavity walls. If the exciton-photon interaction is strong they will form a coupled state called an exciton-polariton. Even if the semiconductor is in its ground state it will surround by a cloud of virtual photons produced by vacuum fluctuations.

B. Quantitative Description of Light Matter Interactions

In this section a mathematical representation of the system is derived in k-space using the Hamiltonian formalism. Simulations based on the observables calculated are used to describe different phenomena present in light matter interactions.

1. *The Hamiltonian*

A one dimensional system is a Fabry-Perot cavity with perfectly reflecting walls and a slab of semiconductor in the middle. A 2-level atom is analogous, but requires a three dimensional simulation which is not practical for simulation in this thesis. The Hamiltonian of such a system is [4]

$$\hat{H}_{total} = \hat{H}_{semi} + \hat{H}_{field} + \hat{H}_{interaction}. \tag{1}$$

Considering only the possibility that there is either 0 or 1 photons in the cavity and the semiconductor is either in its ground state or its 1^{st} excited state, the basis vectors can be represented as:

$$|e\rangle\,|0,0,0...0,0\rangle = |e\rangle\,|\phi\rangle = |e;\phi\rangle$$

which represents the semiconductor in its excited state and zero photons in all cavity modes;

$$|g\rangle |0,0,0..1...0,0\rangle = |g\rangle |j\rangle = |g;j\rangle$$

which represents the semiconductor in its ground state while the j^{th} cavity mode is excited by a photon but the remaining modes are empty. The free field Hamiltonian can then be written in terms of photon creation and destruction operators $\hat{a}_j^\dagger$ and $\hat{a}_j$, respectively as

$$\hat{H}_{field} = \sum_j \hbar\omega_j \hat{a}_j^\dagger \hat{a}_j, \tag{2}$$

where the infinite vacuum fluctuation energy $\frac{1}{2}\sum_j \hbar\omega_j$ has been omitted. The interaction energy in the electric dipole approximation is

$$\hat{H}_{interaction} = -e\hat{E}\cdot\hat{x} = \hat{E}\cdot\hat{d},$$

where $\hat{d}$ is the electric dipole operator connecting the ground and excited states,

$$\hat{d} = d_{eg}\sigma_+ + d_{eg}^*\sigma_- \tag{3}$$

with d_{eg} being the dipole matrix element between ground and excited states. Raising and lowering operators for the semiconductor states are denoted: $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, $\sigma_e = |e\rangle\langle e|$.

The semiconductor Hamiltonian can be expressed in terms of the operator σ_e which will only be nonzero when acting on the only excited state.

$$\hat{H}_{semi} = \hbar\omega_{eg}\sigma_e \tag{4}$$

where $\hbar\omega_{eg}$ is the energy gap between the ground and excited semiconductor state.

To continue determining the field-semiconductor interaction energy we need to come up with a representation of the quantized electric field in the cavity. The second quantized electric field is

$$\hat{E}(x,t) = i\sum_{j=-\infty}^{\infty}\sqrt{\frac{\hbar\omega_j}{\varepsilon_o V}}\left[\hat{a}_j e^{ik_j x} - \hat{a}_j^\dagger e^{-ik_j x}\right]$$

In a cavity with perfectly reflecting walls the electric field at x=0 and x=L is zero. The angular frequencies of the modes are thus

$$\begin{aligned}\omega_n &= k_n c \\ &= \frac{n\pi c}{L}\end{aligned}$$

where n is some positive integer. This leads to the modes having an equal spacing of $\Delta = \frac{\pi c}{L}$.

In general the cavity can hold any mode with frequency ω_n, but in numerical calculations only near-resonant modes will be considered. We take the nearest to resonant mode frequency to be $\omega_o = \omega_{eg} + \delta$, where δ represents the de-tuning from the semiconductor resonance frequency. The nth frequency can now be denoted in terms of the near-resonant mode j_o as $n = j_o + j$. A useful expression involving the difference of the jth mode frequency and the semiconductor resonant frequency is:

$$\omega_j - \omega_{eg} = j\Delta + \delta \tag{5}$$

Summing the k_j and $-k_j$ modes of the electric field, we can now write

$$\begin{aligned} \hat{E}(x,t) &= -2\sum_{j=0}^{\infty} \sqrt{\frac{\hbar\omega_j}{\varepsilon_o V}} \left(\hat{a}_j + \hat{a}_j^\dagger\right) \sin\left[(j_o + j)\,\frac{\pi x}{L}\right] \\ &= \sum_{j=0}^{\infty} C_j \left(\hat{a}_j + \hat{a}_j^\dagger\right) \sin\left[(j_o + j)\,\frac{\pi x}{L}\right] \end{aligned} \tag{6}$$

With this electric field we can now determine the interaction Hamiltonian for the system. By combining equations (3) and (6) in the manner described above we get

$$\begin{aligned} \hat{H}_{interaction} &= \sum_j C_j \left(\hat{a}_j + \hat{a}_j^\dagger\right) \sin\left[(j_o + j)\,\frac{\pi x_s}{L}\right] \cdot \left(d_{eg}\sigma_+ + d_{eg}^*\sigma_-\right) \\ &= \sum_j C_j \left(d_{eg}\hat{a}_j\sigma_+ + d_{eg}^*\hat{a}_j\sigma_- + d_{eg}\hat{a}_j^\dagger\sigma_+ + d_{eg}^*\hat{a}_j^\dagger\sigma_-\right) \sin\left[(j_o + j)\,\frac{\pi x_s}{L}\right] \\ &\simeq \sum_j \hbar \left(\Omega_j\hat{a}_j\sigma_+ + \Omega_j^*\hat{a}_j^\dagger\sigma_-\right) \sin\left[(j_o + j)\,\frac{\pi x_s}{L}\right]. \end{aligned} \tag{7}$$

In the last line of the above equation I have neglected the non-energy conserving terms $\hat{a}_j\sigma_-$ and $\hat{a}_j^\dagger\sigma_+$ by making the rotating wave approximation and defined the constants $\Omega_j = -2d_{eg}\sqrt{\frac{\omega_j}{\varepsilon_o\hbar V}}$ and x_s which is the position of the semiconductor. Adding the individual Hamiltonians (2), (4) and (7) we get the total Hamiltonian for the system gives

$$\hat{H}_{total} = \hbar\omega_{eg}\sigma_e + \sum_j \hbar\omega_j\hat{a}_j^\dagger\hat{a}_j + \sum_j \hbar \left(\Omega_j\hat{a}_j\sigma_+ + \Omega_j^*\hat{a}_j^\dagger\sigma_-\right) \sin\left[(j_o + j)\,\frac{\pi x_s}{L}\right]. \tag{8}$$

A useful next step is to express (8) in matrix form for easier computational manipulation. By representing the basis vectors in terms columns matrices we can construct a square matrix to represent the total Hamiltonian.

$$|e,\phi\rangle = \begin{pmatrix} 1 \\ 0 \\ . \\ . \\ 0 \end{pmatrix}, |g,-n\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ . \\ 0 \end{pmatrix}, |g,-n+1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ . \\ 0 \end{pmatrix}$$

$$\hat{H}_{total} = \hbar \begin{pmatrix} 0 & . & f_{-1} & f_0 & f_1 & . \\ . & . & 0 & 0 & 0 & 0 \\ f_{-1}^* & 0 & -\Delta+\delta & 0 & 0 & 0 \\ f_0^* & 0 & 0 & \delta & 0 & 0 \\ f_1^* & 0 & 0 & 0 & \Delta+\delta & 0 \\ . & 0 & 0 & 0 & 0 & . \end{pmatrix} \tag{9}$$

Where f_j=$\Omega_j \sin\left[(j_o+j)\,\frac{\pi x_s}{L}\right]$ and the Hamiltonian is shifted by the semiconductor energy $\hbar\omega_{eg}$ and then simplified by using Eq.(5) .

2. The Observables

The next step is to determine the semiconductor and photon excitation amplitude, energy density and number density for the system.

a. Probability Amplitudes and Electric Field Squared For general initial conditions we have

$$|\Psi(0)\rangle = A(0)\,|e;\phi\rangle + \sum_j B_j(0)\,|g;j\rangle \tag{10}$$

Where A(0) and B(0) are the mode amplitudes. At any given time then the state vector will have the form

$$|\Psi(t)\rangle = A(t)\,|e;\phi\rangle + \sum_j B_j(t)\,|g;j\rangle\,. \tag{11}$$

Since we already have the Hamiltonian in matrix from it is straightforward to solve for its eigenvalues and eigenvectors. One can express the qth eigenvector of the Hamiltonian as

$$|E^q\rangle = d_e^q\,|e;\phi\rangle + \sum_j d_{g,j}^q\,|g;j\rangle \tag{12}$$

which satisfies the equation $\hat{H}_{total}\left|E^q\right\rangle = E^q\left|E^q\right\rangle$, where E^q is the eigenvalue of the qth eigenstate $\left|E^q\right\rangle$. The coefficients d_e^q and $\mathrm{d}_{g,j}^q$ denote the strength of a single modes contribution to the normal mode. In order to determine the time evolution of the initial state we project the initial conditions onto the eigenstate

$$\left|\Psi\left(0\right)\right\rangle = \sum_q \left|E^q\right\rangle \left\langle E^q|\Psi\left(0\right)\right\rangle$$

Now, the time evolution of the eigenstate is the usual $e^{-i\frac{E^q t}{\hbar}}$ so we get the time dependant state equation

$$\begin{aligned}
\left|\Psi\left(t\right)\right\rangle &= \sum_q e^{-i\frac{E_q t}{\hbar}} \left|E^q\right\rangle \left\langle E^q|\Psi\left(0\right)\right\rangle \\
&= \sum_q e^{-i\frac{E_q t}{\hbar}} \left[d_e^{q*} A\left(0\right) + \sum_j d_{g,j}^{q*} B_j\left(0\right)\right] \left[d_e^q \left|g;\phi\right\rangle + \sum_j d_{g,j}^q \left|g;j\right\rangle\right] \\
&= \sum_q e^{-i\frac{E_q t}{\hbar}} K_q \left[d_e^q \left|e;\phi\right\rangle + \sum_j d_{g,j}^q \left|g;j\right\rangle\right] \qquad (13)
\end{aligned}$$

where I combined the constants together such that $K_q = d_e^{q*} A\left(0\right) + \sum_j d_{g,j}^{q*} B_j\left(0\right)$ which is completely in terms of known initial conditions. Comparing equations (11) and (13) we get the semiconductor excitation amplitude A(t) and the photon excitation amplitude B(t).

$$\begin{aligned}
A\left(t\right) &= \sum_q e^{-i\frac{E_q t}{\hbar}} d_e^q K_q \qquad (14) \\
B_j\left(t\right) &= \sum_q e^{-i\frac{E_q t}{\hbar}} d_{g,j}^q K_q
\end{aligned}$$

As is usual the expectation value of the electric field is zero, so in order to get any information on the electric field we must calculate the expectation value of the square of the field. From (6) we get

$$\begin{aligned}
\hat{E}^2(x,t) &= \sum_{j'} \sum_j C_j C_{j'} \left(\hat{a}_j + \hat{a}_j^\dagger\right)\left(\hat{a}_{j'} + \hat{a}_{j'}^\dagger\right) \sin\left[\left(j_o + j\right)\frac{\pi x}{L}\right] \sin\left[\left(j_o + j'\right)\frac{\pi x}{L}\right] \\
&= \sum_{j,j'} C_j C_{j'} \left(\hat{a}_j \hat{a}_{j'} + 2\hat{a}_j^\dagger \hat{a}_{j'} + \hat{a}_j^\dagger \hat{a}_{j'}^\dagger + \delta_{j,j'}\right) \sin\left[\left(j_o + j\right)\frac{\pi x}{L}\right] \sin\left[\left(j_o + j'\right)\frac{\pi x}{L}\right] \qquad (15)
\end{aligned}$$

Where the commutation relation $\left[\hat{a}_j, \hat{a}_{j'}^\dagger\right] = \delta_{j,j'}$ was used to simplify the final expression.

The expectation value can then be determined as

$$\langle \Psi(t)| \hat{E}^2(x,t) |\Psi(t)\rangle = \sum_j C_j^2 \sin^2\left[(j_o+j)\frac{\pi x}{L}\right] + 2\left|\sum_{j,q} C_j d_{g,j}^q K_q e^{\frac{iE_q t}{\hbar}} \sin\left[(j_o+j)\frac{\pi x}{L}\right]\right|^2$$
$$\simeq 2\left|\sum_{j,q} C_j d_{g,j}^q K_q e^{\frac{iE_q t}{\hbar}} \sin\left[(j_o+j)\frac{\pi x}{L}\right]\right|^2 \quad (16)$$

where the infinite term $\sum_j C_j^2 \sin^2\left[(j_o+j)\frac{\pi x}{L}\right]$ is dropped from the final expression because it gives no useful information about the system.

b. Energy Density The energy density which will be determined here can be more accurately described as the coarse grained energy density, which is the energy density carried by the photons when it is averaged over a volume $l^3 << V$ [5]. The coarse grained energy is described as

$$\hat{H}_T(x) = \hat{H}_E(x) + \hat{H}_M(x) \quad (17)$$
$$\hat{H}_E(x) = \hat{E}(x)\cdot\hat{D}(x) \quad (18)$$
$$\hat{H}_M(x) = \frac{1}{\mu_o}\hat{B}^2(x)$$

Where $\hat{H}_T(x)$ is the total energy and $\hat{H}_E(x)$ & $\hat{H}_M(x)$ represent the energy due to electric and magnetic contributions, respectively. $\hat{E}(x)$ is the electric field and $\hat{D}(x)$ is the electric displacement field. $\hat{B}(x)$ is the magnetic field of the electromagnetic wave. Further definitions of the operators are

$$\hat{D}(x) = \varepsilon_o \hat{E}(x) + \hat{P}(x)$$
$$\hat{P}(x) = -p\widehat{\sigma}(t_o)\delta_{x_A}$$
$$\widehat{\sigma}(t_o) = A(t_o)A^*(t_o)\left[|e\rangle\langle g| + |g\rangle\langle e|\right]$$

Where $\hat{P}(x)$ is the polarization due to the presence of the semiconductor, p represents the polarization strength, and $A(t_o)$ is the semiconductor excitation amplitude described in (14). Evaluating the dot product in equation (18) gives $\hat{H}_E(x) = \varepsilon_o \hat{E}^2(x) - p\widehat{\sigma}(t_o)\hat{E}(x)\delta_{x_A}$, and the known relation between electric and magnetic waves determined by Maxwells equations is $\hat{B}^2(x) = \frac{1}{c^2}\hat{E}^2(x)$. Combining all these results gives the coarse grained energy density as

$$\left\langle \hat{H}(x,t_o)\right\rangle = 2\varepsilon_o \left\langle \hat{E}^2(x)\right\rangle - p\delta_{x_A}\left\langle \widehat{\sigma}(t_o)\cdot\hat{E}(x)\right\rangle \quad (19)$$

Using the same basis as before, $|\Psi(t)\rangle = A(t)|e;\phi\rangle + \sum_j B_j(t)|g;j\rangle$ (13), the expectation value for the electric field squared is the same, $\left\langle \hat{E}^2(x) \right\rangle = 2\left|\sum_j C_j B_j(t) \sin\left[(j_o+j)\frac{\pi x}{L}\right]\right|^2$ (16). The calculation of the remaining expectation goes as follows.

$$\left\langle \widehat{\sigma}(t_o)\cdot\hat{E} \right\rangle = A(t_o)A^*(t_o)\left[\sum_j C_j A^*(t_o)B_j(t_o)\sin[(j+j_0)\frac{\pi x}{L}] + \sum_j C_j A(t_o)B_j^*(t_o)\sin[(j+j_0)\frac{\pi x}{L}]\right]$$
$$= |A(t_o)|^2 \sum_j C_j \left(A^*(t_o)B_j(t_o) + A(t_o)B_j^*(t_o)\right)\sin[(j+j_0)\frac{\pi x}{L}]$$

So the coarse grained energy density at some time t_o is

$$\left\langle \hat{H}(x,t_o) \right\rangle = 4\varepsilon_o \left|\sum_j C_j B_j(t_o)\sin\left[(j_o+j)\frac{\pi x}{L}\right]\right|^2 \tag{20}$$
$$- p\delta_{x_A}|A(t_o)|^2 \sum_j C_j \left(A^*(t_o)B_j(t_o) + A(t_o)B_j^*(t_o)\right)\sin[(j+j_0)\frac{\pi x}{L}] \tag{21}$$

c. Number Density The number density is defined as the density of photons in the cavity normalized with respect to the sum of both the semiconductor and field excitations. Based on this we have the number density $N(x,t)$ being

$$N(x,t) = \frac{\left|\sum_j B_j(t)\sin\left[(j_0+j)\frac{\pi x}{L}\right]\right|^2}{\int_0^1 \left[\left|\sum_j B_j(t)\sin\left[(j_0+j)\frac{\pi x}{L}\right]\right|^2 + \delta_{x_A}|A(t)|^2\right]dx} \tag{22}$$

III. RESULTS AND DISCUSSION

A. Polariton Modes

The polariton system has split energy levels for the cavity modes as is expected. This is shown in Figure 3 where 199 cavity modes are considered, resonant cavity mode $j_o = 10000$ and the coupling strength $\Omega = \frac{5}{\sqrt{j_o}}$.

Physical, essentially one dimensional, systems have been designed using doped quantum wells [7]-[10] which reveal the splitting of the energy levels, as is evident in Figure 3.

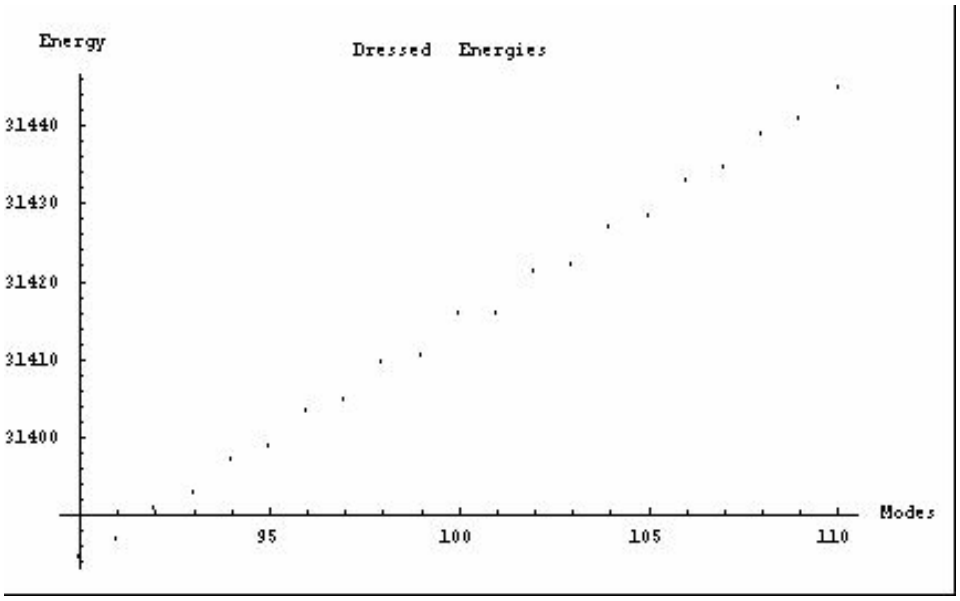

FIG. 3: Energy levels surrounding the resonant mode of the semiconductor. The energies are split above and below the resonant energy.

B. Time Reversal

In this section I will plot the results using Mathematica(see Appendix) in a context that reveals the TR aspect of the system. For calculation purposes I will use units of length such that $\frac{L}{c} = 1$ and place the semiconductor in the center of the cavity (ie. $x_s = \frac{L}{2}$). The following results are determined considering 99 cavity modes, resonant mode $j_o = 10000$, de-tuning $\delta = 0$, and coupling strength $\Omega = \frac{4}{\sqrt{j_o}}$.

Figures 4 and 5 are plots concerning emission by the semiconductor which will be used to compare two scenarios of re-absorption of the photon by the semiconductor, where $|A|$ is given by Eq. (14) and $|E|^2$ by Eq. (15).

Now if we allow the photon to simply reflect off the cavity boundary and get re-absorbed by the semiconductor the spectrum is as shown in Figures 6 and 7.

If instead of simply reflecting the photon on the cavity walls we time reversed the field in a similar manner as done by Yanik [3], the result is shown in Figures 8 and 9.

Consider first the case described in Figure 7 where the photon was reflected off the cavity walls and re-absorbed by the semiconductor. Looking at the semiconductor excitation amplitude shown in Figure 6 we can see that only around 75% of the field was re-absorbed

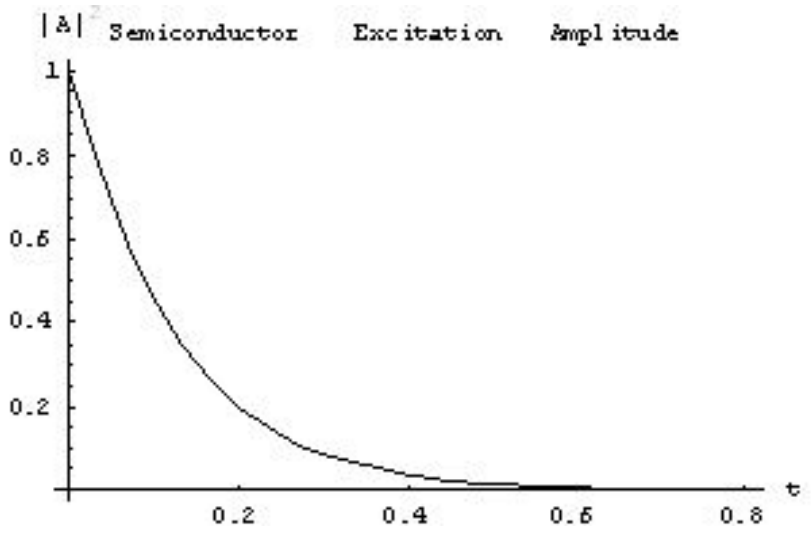

FIG. 4: Semiconductor Excitation Amplitude for the Emission Process

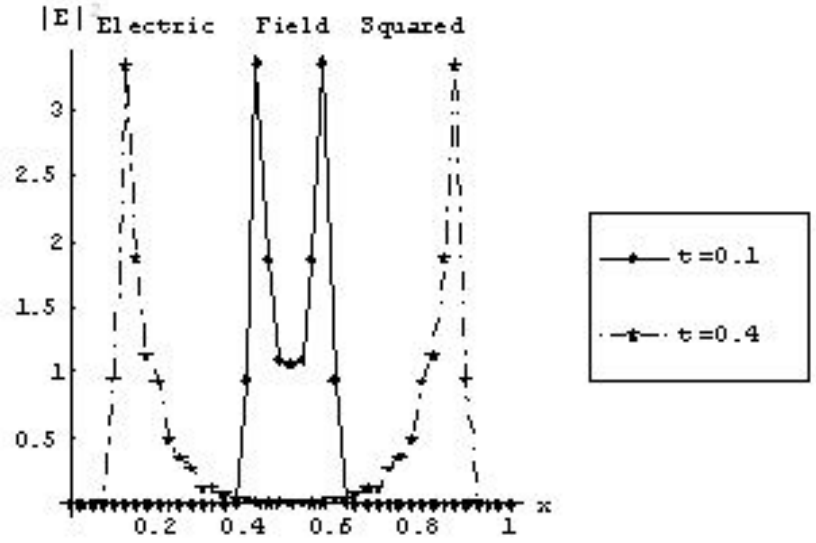

FIG. 5: Electric Field Squared of the Photon emitted by the Semiconductor

by the semiconductor, and it can also be noted that the re-emission that immediately occurs is asymmetric. Comparing this with the time reversed photon shown in Figure 9 we can see that the wave shape looks considerably different as the photon approaches the semiconductor. The field in the TR case is identical to the field shown in Figure 5, which is the field of the emitted photon, except that it is propagating in the opposite direction. As a result of

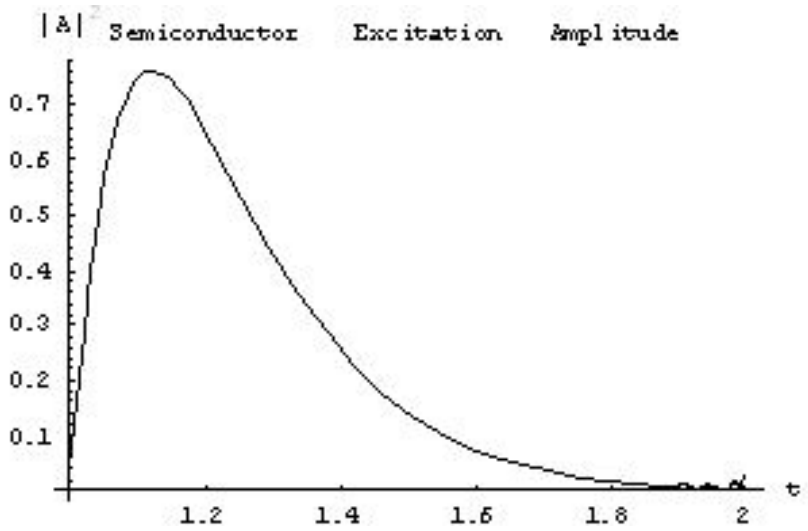

FIG. 6: Semiconductor Excitation Amplitude of the absorption and re-emittance of a photon reflected at the cavity boundary.

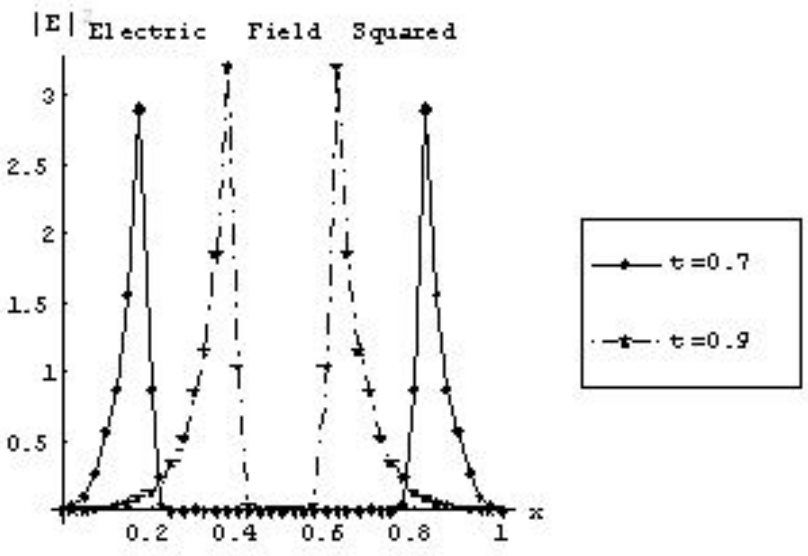

FIG. 7: Electric Field squared of the Photon Reflected by the Cavity Boundary.

this symmetry the entire photon field is re-absorbed by the semiconductor before being immediately re-emitted, as can be seen in Figure 8. From this we can conclude that the TR photon was focused to a tighter focal spot size then the reflected photon.

Since we want to know if the photon can be localized I will look at the electric field of the photon for extremely short times during the emission process. The result is shown in

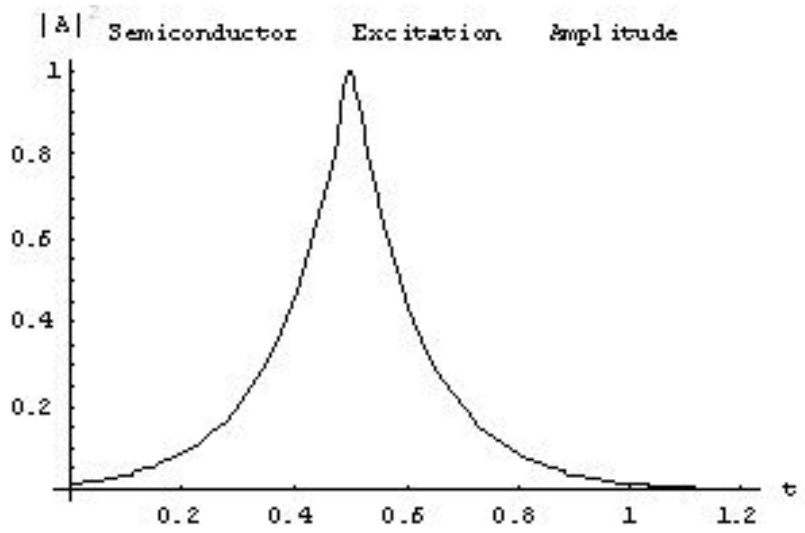

FIG. 8: Semiconductor Excitation Amplitude of a Photon TR at the cavity Boundary and then re-absorbed and emitted.

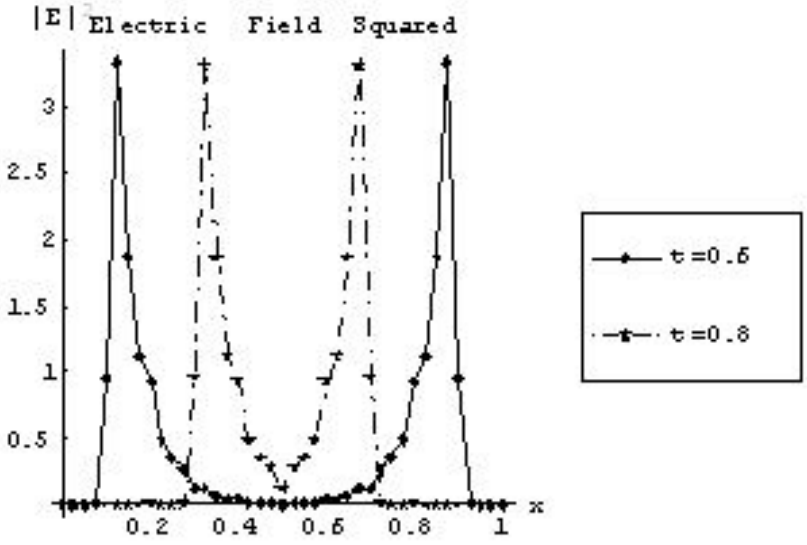

FIG. 9: Electric Field squared of the TR photon approaching the semiconductor

Figure 10 with 299 available cavity modes considered.

It was mentioned earlier that if all the components of the field are included simultaneously then the photon could be shown to be localized. My model one consists of one dimension, so there is only one component of the field and it was found that the photon is localized. At times a lot shorter then the decay rate of the semiconductor, it is seen in Figure 10

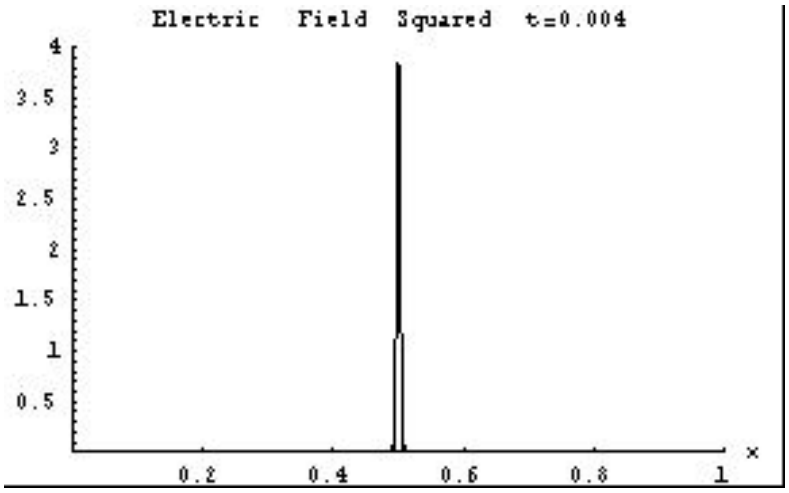

FIG. 10: Electric Field squared of emission by the semiconductor for times a lot shorter then the decay rate

that there exists a non-zero non- propagating field. It can also be shown that as the number of modes considered in the simulation are increased, the field at the position of the semiconductor approaches a delta function. This indicates that for short times before the photon is completely emitted, it is localized to the size of the atom; at least in one dimension.

C. Photon Cloud, Semiconductor in Ground State

The following plots are generated using essentially the same Mathematica program that was used in the time reversal section, except that the photon and excitation energies add (which is in contrast with Eq. 5), and where the energy and number densities are used to describe the system. Starting the system out with the semiconductor in the ground state and with all of the 99 considered photon modes empty, the energy density at time $t = 5$ is displayed in Figure 11.

The number density for the system described above was found to produce the same plot as the energy density and is thus not necessary to show. The Energy density shown in Figure 11 is interesting because it shows that for a system in a vacuum virtual photons are created in the surrounding region which resonantly exchange energy with the semiconductor (ie. Rabi Oscillations). The two bumps around the semiconductor indicate an accumulation of photons which "dress" the semiconductor in what is referred to as a photon cloud. The

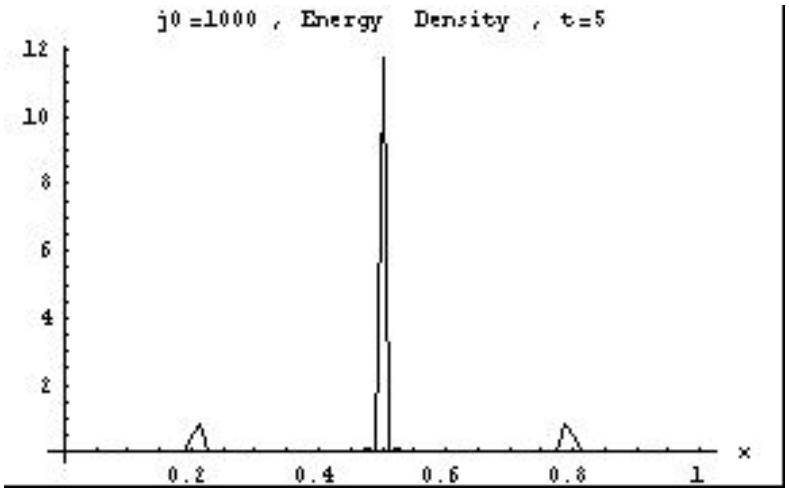

FIG. 11: Energy Density of a Photon Cloud. System started out with semiconductor in the ground state and zero photon states.

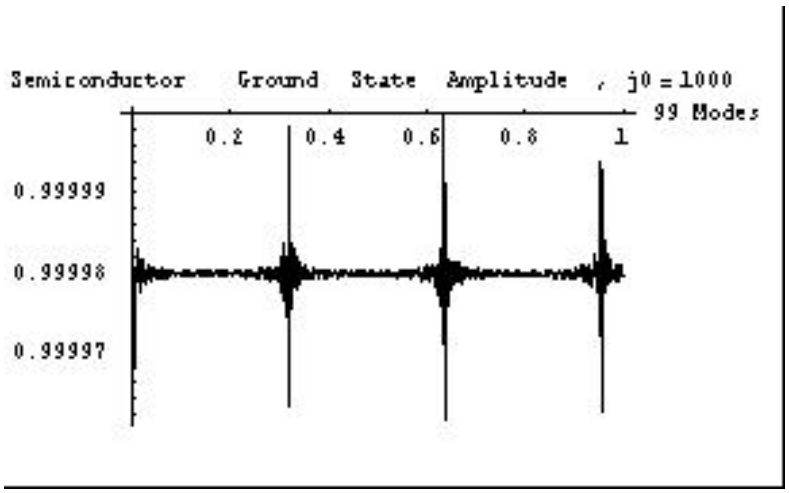

FIG. 12: Probability Amplitude for the initially ground state semiconductor to be in the ground state, where there was initially zero photons in the cavity.

fact that the number density has the same shape as the energy density is due to the small variation in the photon frequency with respect to the resonant mode. Thus the differing factor of $\sqrt{\hbar\omega}$ has little visible effect on the density. It should be no surprise that they have very similar shapes since the energy density is essentially solely due to photon contributions.

The probability amplitude of the semiconductor being in the ground state, with the same initial conditions as above, is shown in Figure 12.

The collapse and revivability of the semiconductor ground state shown in Figure 12 is an interesting result that again shows an oscillation between the photon and exciton states in this ground state system.

---

[1] J. de Rosny & M. Fink "Overcoming the Diffraction Limit in Wave Physics Using a Time-Reversal Mirror and Novel Acoustic Sink" Phys. Rev. Lett. vol.89 p.124301

[2] R. Dorn, S. Quabis and G. Leuchs "Focusing a radially polarized light beam to a significantly small spot size" Opt. Comm. 179(2000)

[3] Mehmet Yanik and Shanhui Fan "Time Reversal of Light with Linear Optics and Modulators" Phys. Rev. Lett. vol.93 p.173903

[4] Martin Ligare and Ryan Oliveri "The calculated photon: Visualization of a quantum field" Am. J. Phys. 70(1) 2002

[5] R. Passante "Cloud of virtual photons in the ground state of the hydrogen atom" Phys. Rev. A Vol. 31 p.2827

[6] G. Compagno, R. Passante and F. Persico "Atom-Field Interactions and Dressed Atoms" Ch.5

[7] Stanely Pau et al. "Microcavity exciton-polariton splitting in the linear regime" Phys. Rev. B Vol. 51 p. 14 437

[8] Simone De Liberato and Cristiano Ciuti "How the coupling to the vacuum field of a microcavity changes the nature of a conduction electron in a doped quantum well" Quant-Ph 28 Feb 2008

[9] C. Weisbuch et al. "Observation of the Coupled Exciton-Photon Mode Splitting in a Semiconductor Quantum Microcavity" Phys. Rev. Lett. Vol. 69 p.3314

[10] V. Savona, Z. Hradil and A. Quattropani "Quantum theory of quantum-well polaritons in semiconductor microcavities" Phys. Rev. B Vol. 49 p.8774

IV. APPENDIX

(*Time Reversed Atom around j0*)

Clear[nmodes, dim, Ω, xa, Δ, j0, δ, h, vals, vecs, atomamp, photonamp, e2, x, t, j, l, xmin, xmax, np, dx, Con, s, A0]

```
<< Graphics`MultipleListPlot`

nmodes = 99; (*number of modes to consider*)
dim = nmodes + 1; (*Dimension of the hamiltonian matrix*)
Δ = π; (*mode spacing*)
j0 = 10000; (*nearest to resonant mode*)
δ = 0; (*detuning*)
Ω = 4/(j0)^(0.5); (*coupling strength*)
t0 = 0; (*time to Time Reverse*)
xa = 0.5; (*atom position*)

(*Hamiltonian*)

H = Table[0, {i, dim}, {j, dim}];
Do[{H[[1, j]] = H[[j, 1]] = Ω * (j0 - dim /2 - 1 + j)^(0.5)Sin[(j0 - dim /2 + 1 + j) * xa * π],
H[[j, j]] = (- dim /2 - 1 + j) * Δ + δ},
{j, 2, dim}];

{vals, vecs} = Eigensystem[H];

(*Constants*)
A0 = (∑_{k=1}^{dim} Exp[-i * vals[[k]] * t0] * vecs[[k, 1]]^2);
Con[s_] =
(Conjugate[vecs[[s, 1]]] * A0+
∑_{j=-(nmodes-1)/2}^{(nmodes-1)/2} Conjugate[vecs[[s, dim /2 + 1 + j]]]*
(∑_{k=1}^{dim} Exp[-i * vals[[k]] * t0] * Conjugate[vecs[[k, 1]]] * vecs[[k, dim /2 + 1 + j]]));
(*Semiconductor Probability amplitude*)
semiamp[t_] = ∑_{q=1}^{dim} Exp[-i * vals[[q]] * t] * vecs[[q, 1]] * Con[q];
Plot[Abs[semiamp[t]], {t, 0, 0.8}, PlotRange → All, PlotLabel →
"Semi Excitation Amplitude", AxesLabel → {"t", "|A|^2"}];
```

```
(*Cavity mode Probability amplitude*)
photonamp[t_, j_]:= Sum[Exp[-i * vals[[q]] * t] * vecs[[q, dim/2 + 1 + j]] * Con[q], {q, 1, dim}];
(*Electric Field Squared*)
e2[x_, t_] = (Abs [Sum[photonamp[t, l] * Sin[(j0 + l) * π * x], {l, -(nmodes-1)/2, (nmodes-1)/2}]]) ^2;
(*Plot*)
xmin = 0;
xmax = 1;
time = 0.1;
np = 40;
dx = (xmax - xmin)/np;
MultipleListPlot[{Table[{x, e2[x, time]}, {x, xmin, xmax, dx}],
Table[{x, e2[x, 0.4]}, {x, xmin, xmax, dx}]}, PlotJoined → True,
PlotRange → All, PlotLabel → "Electric Field Squared",
PlotStyle → {GrayLevel[0], Dashing[{Dot, Dash}]},
PlotLegend → {"t=0.1", "t=0.4"}, AspectRatio → 1, AxesLabel → {"x", "|E|^2"}];
```